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AUTHOR Hall, Dennis R.
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ABSTRACT

Rhetoric and mathematics have much in common that can help explain the composing process. Common elements of rhetoric and mathematics important to the teaching of writing are (1) relationships between syntax and semantics, (2) practices of representation, and (3) focus on problem solving. Recent emphasis on "repair processes" in mathematics is similar to rhetoric's concern for the processes of revision and drafting. The artificiality of algebra maintains a distinction between the language that speaks directly about objects and the language in which the object language is spoken about. Rhetoric, on the other hand, tends to blur this language/metalanguage distinction. However, by applying mathematical logic to the writing process and using mathematical language in the writing class, the distinction can be clarified. Though both rhetoric and mathematics are means of knowing, of discovery, they mainly seek solutions to solve problems and communicate effectively. In the classroom, treating rhetoric as mathematics could provide an antidote to some of the "natural language" biases that plague the contemporary writing class. It may also encourage a problem-solving approach to the composing process. Finally, it may earn status for rhetoric by associating it with an idiom common in the natural and social sciences--areas that have established credibility. (Figures illustrating the relationship between rhetoric and mathematics are appended.) (SRT)

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Dennis R. Hall
English Department
University of Louisville
Louisville, KY 40292
(502) 588-6801

Mathematical Approaches to the Composing Process

As have so very many others, I grew up and was educated under the influence of the "alibi of mathematical ineptitude." I am sure many of you are familiar with this license for institutional sloth and individual self indulgence. It holds that large chunks of humanity, otherwise thought capable of pretty sophisticated learning, simply cannot do mathematics at levels much beyond the arithmetic. Young Hall, it was routine to note, does not exhibit an "aptitude" for mathematics; it really is not his bent, for he cannot think mathematically. Expectations under this alibi are commonly met as well as conveniently justified for all concerned. I marginally negotiated the minimum standards that a sense of academic decency in the late 50's would allow, but thereafter it was thought neither necessary nor useful to cloud my consciousness with a mathematical thought.

A stint as associate dean, a computer science course, and some reading in cognitive psychology have since lead me to embrace the occasional mathematical thought, apparently without scandal or injury to myself or others. While most of my colleagues and many of my students are far better schooled in mathematics than I will ever be, even this

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limited experience demonstrates that just about anyone on the face of the earth is capable of something resembling mathematical thought, if one can be rid of the alibi of mathematical ineptitude. Though less astounding, this limited experience has also made me acutely aware of some theoretical similarities between mathematical and rhetorical concepts and operations, while recent experience teaching second-term freshmen in the Writing Across the Curriculum (WAC) mode has suggested some possible practical benefits of making these similarities an explicit part of the approach to the writng process in such a course.

*

To remain either substantially unlettered or unnumbered is simply not going to serve the educational processes of individuals or institutions in the remainder of this century. For some time we have acknowledged, if not recognized in behavior, the dangers of "verbal" and "quantitative" specialist categories. We have long heard about risks of neglecting the "verbal" to the specialist in the "quantitative" realms of knowledge, and more recently we have become aware of the dangers to the specialist in the "verbal" who excludes the "quantitative." Yet these capacities are more intricately interwoven, functionally as well as theoretically more interdependent than even these warnings acknowledge.

Increasingly, the distinction itself is beginning to dissolve. Consider, for example, that the syllabus of almost all logic courses now evenly divides its attention between a categorical logic worthy of Aristotle and symbolic logic characteristic of Bertrand Russell or Boolean algebra. Even that bastion of conservatism the Graduate Record Examination now reports an "analytical" as well as "verbal" and "quantative" scores, in an effort somehow to deal with the awkwardness of its taxonomy of aptitudes. If, as good post-modernists, we can no longer resolutely divide fiction from non-fiction, the knower from the known, experience from analysis, the descriptive from the prescriptive, theory from praxis, structure from meaning, quantity from quality--to mention only those pairs which immediately leap to mind--then there is not much percentage in sustaining a separation of those skills identified, however awkwardly, by the language arts on the one hand and the mathematical arts on the other.

While significant differences separate the enterprise we call rhetoric from that we call mathematics, both undertakings have a great deal in common as cognitive exercises, as manifestations of as well as mechanisms for what Max Wertheimer has called Productive Thinking (New York: Harper, 1945, 1959, 1982). These elements common to the study and practice of mathematics and rhetoric are

useful to an understanding of the composing process in the writing classroom and are helpful in marketing a process appraoch to many students.

* *

(#1) Retoric and mathematics, more immediately and more acutely than do most academic disciplines, involve people in the rewards and the dangers of an extensive series of delicate balances. Indeed current critical theory would describe this involvement not so much in terms of the stasis of balance as in terms of the dynamics of reciprocation, as controlled and, it is hoped, productive oscillation:

-- between the abstract and the concrete

-- between theory and praxis

-- between rule and license

-- between convention and invention

-- between given and new information

-- between discovery and making

-- between recreation and creation

-- between formula and form

-- between coherence and incoherence

-- between cognitive assonance and dissonance

-- between relations and operations

-- between natural and the artificial

-- between the diachronic and the synchronic

-- between the syntagmatic and the paradigmatic

-- between language and metalanguage

-- between communicating and knowing

-- between encoding and decoding, or if you will, reading and writing

In the past, teaching and learning in both rhetoric and mathematics was hidebound. Playing it safe, the emphasis fell to the necessay mastery of conventional forms and operations which efficiently dominated the consciousness of both enterprises, leading to rule-governed practices. Writing in a collection called The Development of Mathematical Thinking, Robert B. Davis summarizes recent changes in mathematics, but he might as well have been speaking of now familair developments in rhetoric:

"For most adults in years past, mathematics probably meant memorizing certain specific techniques, and thereafter recalling them and using them as necessary. This is probably still true for many people even today. For a minority of people--including mathematicians, engineers, scientists, computer specialists, and a growing number of people in the health care field, in psychology, in education, in art, and elsewhere--the use of mathematics has nowadays a different quality. For these people, mathematics is an expandable tool for solving novel problems, for which no previously learned algorithm will be entirely sufficient.. . . [people are] exploring new territory and

this often means exploring new mathematical territory, as well. Hence thoughfulness, the rethinking of old assumptions, intelligent guessing, insight, and shrewd planning are all integral parts of the task.

Unfortunately, this "creative" aspect of mathematics has typically been ignored in most school curricula, and the whole idea is foreign to most of the general public.. . .

In recent years an alternative research paradigm has appeared in the world of mathematics education. This alternative paradigm shows considerable promise for increasing the emphasis on creativity, vision, understanding, insight, and the like, while at the same time paying proper attention to rote drill, routine practice, and "meaningless" algorithmic performance.. . .

This approach is beginning to demonstrate the ability to create a serious theory for the analysis of those processes of thinking that are required in dealing with tasks of a mathematical nature.("Complex Mathematical Cognition," in The Development of Mathematical Thinking, ed. Herbert P. Ginsburg. New York: Academic Press, 1983, p. 288.)

Tasks of a mathematical nature are very often tasks of a rhetorical nature, for that matter of a human nature. But the point of greatest interest to the new mathematics as to the new rhetoric is the cognitive status of both enterprises. The functions of computation and communication

have yielded appropriate honor and attention--even place--to discovery and change, to mathematics and to rhetoric as means of knowing as well as means of accomplishing immediate tasks. While a host of common elements are manifest in this epistemic pursuit, perhaps the most important, at least to those of us in the writing classroom, are:

(#2) --relationships between synatx and semantics

--practices of representation

--focus on problem solving

Syntax and Semantics:

Mathematics, to be sure is language, but we may too little apprreciate how well, that is how clearly, it maintains the distinction between the realms of the semantic and the synatactic and their interactions. For example, signs of aggregation-- ( ), { }, [ ], ---- and the like--create semantic constructs, and these are clearly drawn in contrast with signs of operation-- +, -, $\sqrt{}$, x, $\div$, and the like-- which create synatactic contructs. We have definitions of real numbers and polynomials and quadratic equations and we have the laws of addition, multiplication, and distribution.

(#3) **To study the synatx and the semantcs of mathematics at all levels has become the dominant reserach paradigm for investigations into mahtematics cognition and pedagogy in**

recent years. As this diagram of the syntax of substraction illustrates.

(#4) Indeed, protocal analyses and a variety of mapping procedures are common enough to suggest an interest in a "process approach" to the arithmetic and algebraic equivalents to what we call the composing process. And perhaps the clearest evidence of the kinship of our undertakings is the research being done in mathematical "repair processes" which are more than rough equivalents to rhetoric's concern for processes of revision and drafting.

Mathematics, whatever else may be said of it however, maintains distinctions between "properties" and "relationships," on the one hand, and "operations," on the other hand, with greater clarity and consistency than rhetoric is able to achieve. The distinction is important to both rhetoric and mathematics and to productive thinking of either the algorithmic or the creative sort. Curtis Wilson's account is the clearest I am aware of: "The distinction between a relation and an operation is just that a relation holds between two or more elements of a manifold [a many-ness of elements] simply in virtue of <u>what</u> and <u>how</u> the elements are, while an operation is something that <u>we</u> can perform, an action that we may <u>will</u> to carry out.

Equality, for instance, is a relation; addition is an operation" ("Groups, Rings and Lattices," The St. John's Review 36:2 (Spring 1985), 5).

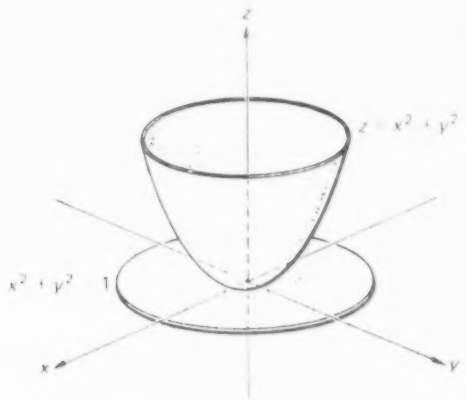
Rhetoric's own attention to syntax and semantics valiently seeks to maintain essentially this same distinction and for the same purposes: to confront efficiently the many splendored thing, to discover its order, and to account for that order in terms of what preceeded it and what will follow it. The relative clarity of the mathematical practice can serve both to illustrate and to focus the distinction for students of rhetoric in their processes of invention and arrangement and even in their more mundane acts of predication on the sentence level.

Representation:

In Computation and Cognition, a book as wild as it is fascinating, Zenon Pylyshyn bluntly notes that "one of the main things cognizers have in common is, they act on the basis of representations (Cambridge, MS:MIT, 1984, p.xii), a point obvious enough I suppose in theory, but commonly overlooked at leats in rhetorical practice. Arnold Glass and James Holyoak, bastions of empirical conservatism, agree, noting that "Knowledge really has two essential aspects, which correspond to the concepts of represntation

and process" (Cognition, 2nd ed. New York: Random House, 1986, p. 5). In an illustration of the distinction between the content and the code in mental representations, Glass and Holyoak point to a salient difference between explanation based in the rhetorical and the mathematical arts:

(#5) [An] example of alternative codes is illustrated in figure 1.2. We might represent this picture by a description in English, such as a "picture of a cup without a handle on a flat saucer."



But as Figure 1.2 illustrates, the picture could also be represented by mathematical equations. The saucer can be represented by $x^2 + y^2 = 1$ (the equation of a circle) and the cup by $z = x^2 + y^2$ (the equation of a paraboloid). Most people probably find the English description more intuitive or simpler than the equations. But if you had to find the volume of a cup, the English code cup would prove far less useful than the equation $z = x^2 + y^2$. This idea is an important one to remember: While different codes can

represent the same information, which code is most useful depends upon what the information has to be used for--how it needs to be processed (p.6-7)."

The artificiality of algebra thus allows one better to understand the language/metalanguage separation---that is, to maintain a sharp distinction between the language that speaks directly about objects and the langauge in which the object language is spoken about---than does the seemingly "natural language" of rhetoric. And the same relative clarity holds for the distinction between relationships and operations, which distinction is a clear analogue to cognitive psychology's distinction between represnetation and process. Algebra, the language of mathematics, far earlier and far more readily than rhetoric applies this notion of code switching, a process essential to cognition,

Problem Solving:

While rhetoric will occasionally abandon itself to the pursuit of truth for its own sake, it is a relatively rare philosophical fit, and the knack, as Plato put it, remains for the most part pragmatic, seeking effectively to communicate, to pursuade. So too, mathematics is capable of flights of fancy worthy of Willaim Blake, but these are narrowly distributed tickets to ride, and the far more common aim is to serve a communicative or persuasive

function. Rhetoric and mathetics are indeed means of knowing, of genuine discovery, but the knowledge is commonly the by-product of some other, more pedestrain end. Problem solving is very much the common heart of the matter.

Here again, the representational flexibility is the feature of mathematics of greatest rhetorical interest. The penchant for, as well as ease of, code switching characteristic of classical as well as modern algebra is crucial to its problem solving power. The advantage of algebra, of cousre, is its ability to escape arithmetic's bondage to the specific instance and to formulate generalizations that will stand the test of specific applciation. And, of course, it allows one systematically to pursue the presently unknown. These adavantages derive from its artifice, as for example its power to represent constants and vaiables by letters and operations by other symbols.

(#6) Mathematical discourse routinely begins with something like, "Let a represent one-half the vertex angle of the cone and let b represent the acute angle formed by the axis of the cone and the intersecting plane" (Charles G. Moore, "Contemporary Conic Sections: The Ellipse and the Lithotripter," Consortium 20 (November, 1986), 3)---a genesis, an act of creation uttered without a moment's concern for usurping the the divine prerogrative.

(#7) As James L. Adams amply demonstrates in Conceptual Blockbusting (2nd ed. New York: Norton, 1980; see esp. chapt. six, "Alternate Thinking Languages," pp.83-100.) representation is essential to productive thinking whether one seeks to break a writer's or a mathematician's block. Selecting the appropriate problem-solving langauge is crucial to discovery. To make such choices, however, one must be sufficiently aware of their existence to look for them. In regard to this multiplicity of representations, mathematics proves a powerful aid to rhetorical invention, sometimes in suggesting an immediate solution but perhaps more often and more effectively in providiing a warrant for continued systematic pursuit within this or other systems of representation---in visual thinking, in other sensory langauges, in binary codes, in a neglected pentad or even in a dog-eared list of Cicero's common topics.

* * *

What, you may reasonably ask, are the practical implications, if any, of these largely theoretical and in the final analysis pretty obvious similarities between the rhetorical and the mathematical enterprsies? Well, treating rhetoric as if it were mathematics in limited but noisily conspicuous ways should serve several purposes:

First, the practice will contribute an antidote to some of the "natural language" biases and alibis which plague the contemporary writing class. Even first year students need a a little critical theory, and algebraic representation is a decent place to start. Create operational symbols. Use the front of the alphabet to mark constant relationships and letters from the end of the alphabet for variables. Diagram ideas or arguments rather than sentences or paragraphs. Create venn diagrams and flow charts for inventive procedures and to represent processes of revision and drafting. But above all describe clear alternative systems of representation and let students see them as such.

And let students see them as aggressively artificial. Point to the benefits as well as the limitations of artifice. Students may in this "mathematical" sooner come to recognize and make use of the difference between experience and analysis, the difference between the concrete and the abstract, and the difference between the language employed in their discourse upon the objects of their attention (language) and the language used in the dicourse about that discourse (metalanguage).

Second, the practice may suggest to one degree or another a problem-solving approach to the composing process: the problem of doing the work, the problem of doing the work well, the problem of the first paper, the problems of

invention and library research, the problems of arrangement, and so forth. Students who find themselves floundering in a sea of sticky ogranicism often grasp relief in representing the composing process to themselves as a sequence of constituent problems to be solved, not unlike the steps in a complex equation. While here there is the risk of falling into routine, for most people complex equations provide adequate opportunity for learning through error and are recursive enough to promote discovery. It is also the case that problem-solving--if the problems are at all challenging--is less likely to fall into the rule-goverend than in plugging either algorithms or rhetorical formulae.

Third, the practice may earn a bit more status for the rhetorical enterprise by its association with an idiom common in the natural sciences and increasingly common in the social sciences. Even the anumerical in academe soon come to the knowledge that mathematics is important, that as an idiom basic to "science," it enjoys special cognitive authority. Rhetoric's mathematics-like character as basic idiom and means of knowing has created for it a new credibility in academe; witness the apparent success of WAC.

(#7) And credibility, Bruno Latour and Steve Woolgar argue, is academe's most important scientific product. "They see scientists as continually engaged in converting 'capital' from one form to another, as 'moves' become

necessary in the scientific field. 'Credibility' is the 'credit' which allows scientists to convert capital: the basic aim of a scientist's strategy is therefore to maximize credibility. Recognition of a recent achievement increases credibility, which can then be 'cashed' in the form of a research grant, converted into equipment (or 'inscription devices') and recruited colleagues, then into data, argumentation, fresh publications, more credibility. . . and so on. Credibility is not itself the reward: it is the credit which makes possible the variety of moves (and hence of rewards, of motives and needs satisfied)("Introduction Part I," Science in Context, Barry Barnes and David Edge, eds. Cambridge, MS:MIT, 1982, pp. 16-17; Bruno Latour and Steve Woolgar, "The Cycle of Credibility," pp. 35-43).

(#9) Presumably the cycle holds for rhetoric and rhetoricians as well as scientists. And through mathematics, then, I suggest rhetoric may construct a more credible and consequently more powerful academic discourse community.

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POLES OF RECIPROCATION

RHETORIC and MATHEMATICS

-- between the abstract and the concrete

-- between theory and praxis

-- between rule and license

-- between convention and invention

-- between given information and the new

-- between discovery and making

-- between recreation and creation

-- between formula and form,

-- between coherence and incoherence.

-- between relations and operations

-- between natural and the artificial

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-- between language and metalanguage

-- between communicating and knowing

-- between encoding and decoding, or if you will,
reading and writing

--relationships between synatx and semantics

--practices of representation

--focus on problem solving

gns of aggregation ( ), { }, [ ], ———, etc

signs of operation +, -, $\sqrt{}$, x, $\div$ etc.

DISTINGUISHING SYNTAX AND SEMANTICS

Written subtraction can be analyzed as an algorithm defined by a set of *syntactic* rules that prescribe how problems should be written, an order in which certain operations must be performed, and which kinds of symbols belong in which positions. Although the syntax may reflect an underlying *semantics*, or meaning, an algorithm need not include any explicit reference to the semantics in order to be successfully performed. Figure 10.1 shows an algorithm for subtraction that is entirely syntactic in nature.

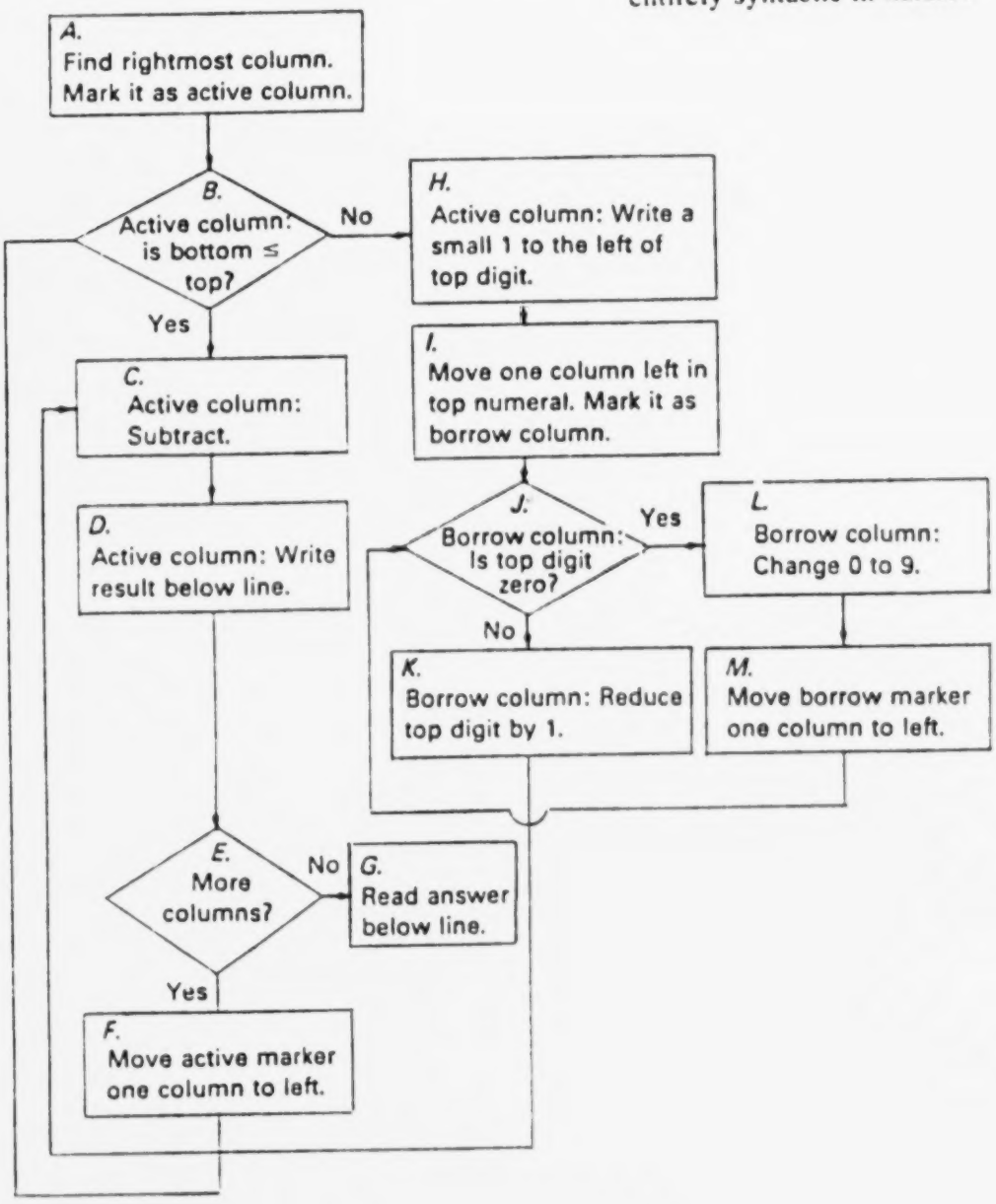

FIG. 10.1. Syntactic algorithm for subtraction.

Lauren B. Resnick, "Syntax and Semantics in Learning to Substract," Addition and Substraction: A Cognitive Perspective, Thomas P. Carpenter, James M. Moser, and Thomas A. Romberg, eds. Hillsdale, NJ: Lawrence Erlbaum, 1982, p. 154.

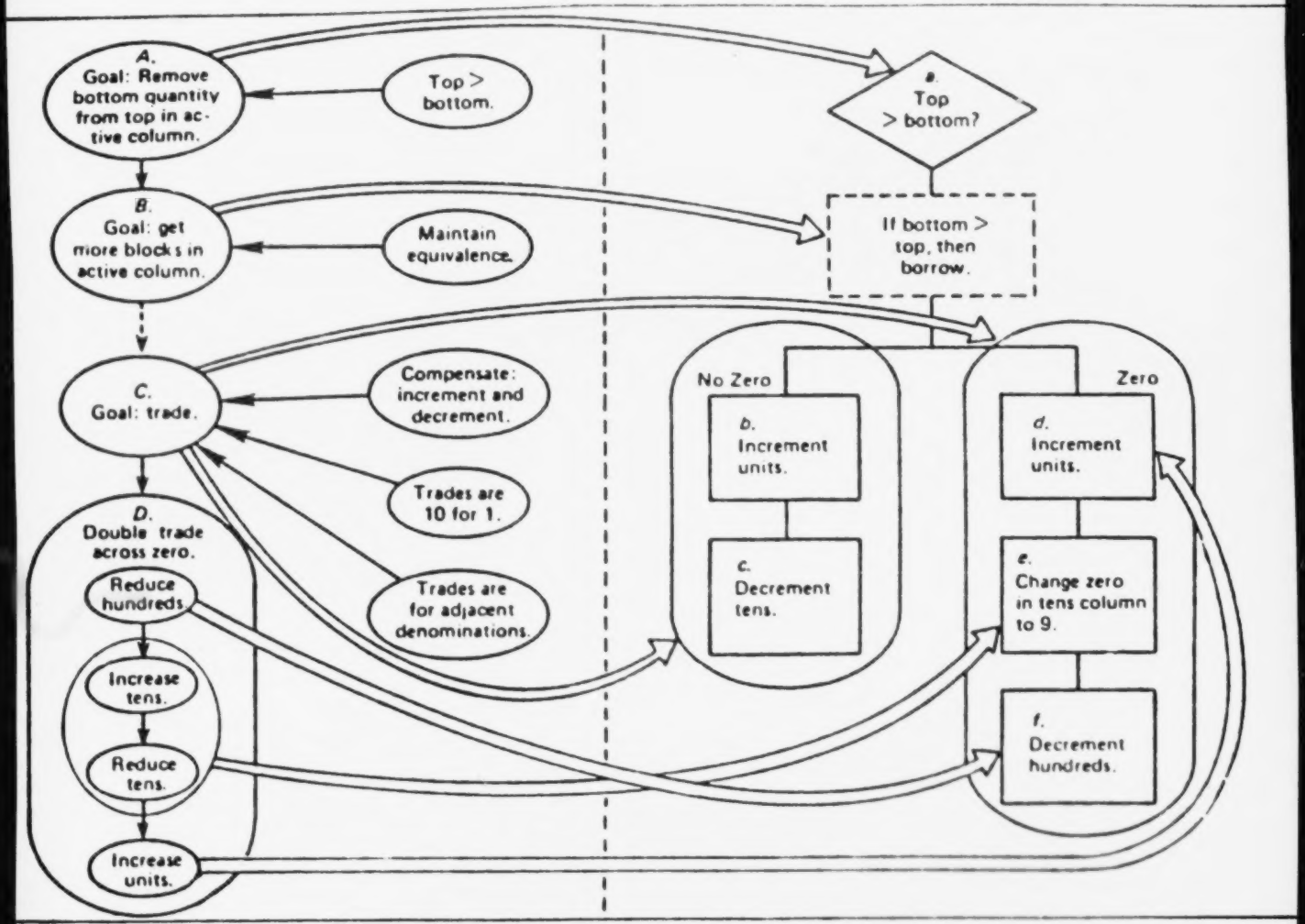

FIG. 10.4. Conceptual links facilitated by mapping

Another example of alternative codes is illustrated in Figure 1.2. We might represent this picture by a description in English, such as a "picture of a cup without a handle on a flat saucer." But as Figure 1.2 illustrates, the picture could also be represented by mathematical equations. The saucer can be represented by $x^2 + y^2 = 1$ (the equation of a circle) and the cup by $z = x^2 + y^2$ (the equation of a paraboloid). Most people probably find the English description more intuitive or simpler than the equations. But if you had to find the volume of the cup, the English code *cup* would prove far less useful than the equation $z = x^2 + y^2$. This idea is an important one to remember: While different

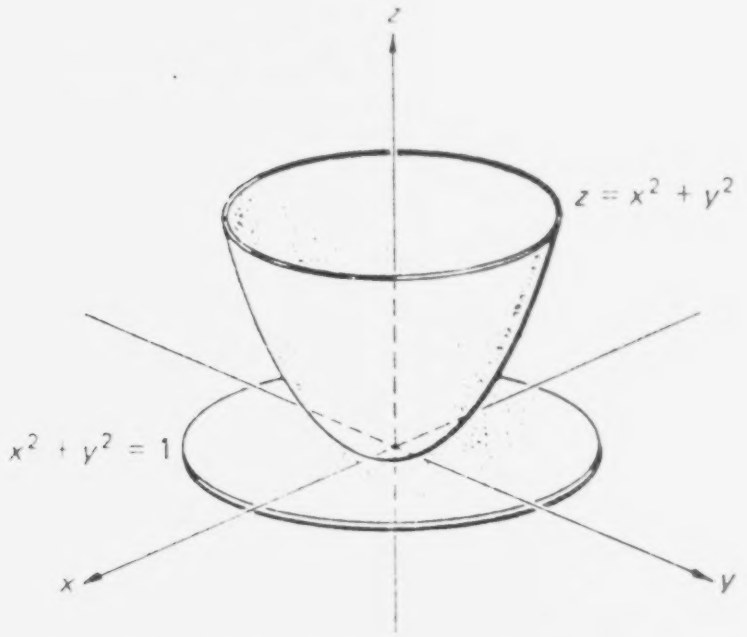

Figure 1.2 Alternative codes. This cup and saucer can be represented by equations.

Contemporary Conic Sections

The Ellipse and the Hyperbola

Charles G. Moore

Northern Arizona University
Flagstaff, AZ 86011

Figure 1.

Most mathematics teachers are fond of the conic sections —the parabola, the ellipse, the hyperbola. The Greek mathematician Apollonius wrote extensively on the conic sections, circa 200 BC, which resulted in 400 propositions [Eves 1969]. Apollonius probably had no idea of the number and variety of practical applications which would ultimately be found for his conic sections, as we will see further into this article. But first, let's briefly review the parabola, ellipse, and hyperbola as sections of a right circular cone cut by a plane.

Let α represent one-half the vertex angle of the cone and let β represent the acute angle formed by the axis of the cone and the intersecting plane. The curve formed by the intersection of the cone by a plane will be:

1) a parabola if $\alpha = \beta$;
2) an ellipse if $\alpha < \beta$; and
3) a hyperbola if $\alpha > \beta$.

Figure 1 illustrates the curves defined above as conic sections.

The curves can also be defined as the path of a moving point as follows:

1) the parabola is the path of a point moving so that its distance from a fixed point, called the focus, is equal to its distance from a fixed line;
2) the ellipse is the path of a point moving so that the sum of its distances from two fixed points, called foci, is a constant;
3) the hyperbola is the path of a point moving so that the difference of its distances from two fixed points, called loci, is a constant.

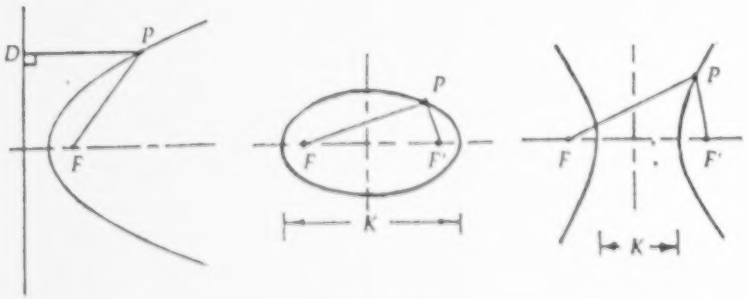

Figure 2.

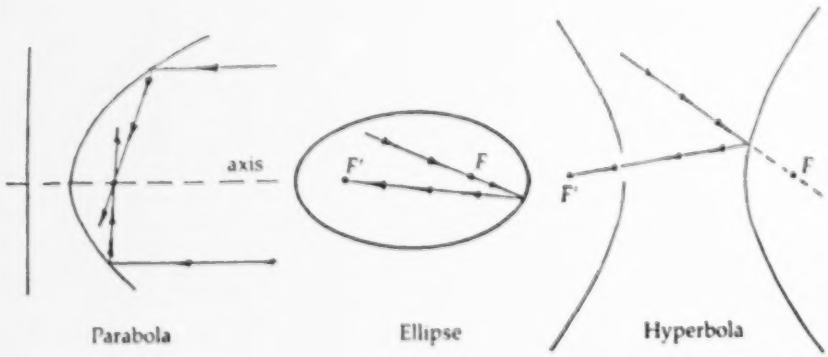

Figure 3.

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INTELLECTUAL BLOCKS RESULT in an inefficient choice of mental tactics or a shortage of intellectual ammunition. Expressive blocks inhibit your vital ability to communicate ideas—not only to others, but to yourself as well. Let us look at the following blocks:

1. Solving the problem using an incorrect language (verbal, mathematical, visual) — as in trying to solve a problem mathematically when it can more easily be accomplished visually
2. Inflexible or inadequate use of intellectual problem-solving strategies
3. Lack of, or incorrect, information
4. Inadequate language skill to express and record ideas (verbally, musically, visually, etc.)

The cycle of credibility

Figure 1 illustrates the cycle of credibility. The notion of credibility makes possible the conversion between money, data, prestige, credentials, problem areas, argument, papers, and so on. Whereas many studies of science focus on one or other small section of this circle, our argument is that each facet is but one part of an endless cycle of investment and conversion. If, for example, we portray scientists as motivated by a search for reward, only a small minority of the observed activity can be explained. If instead we suppose that scientists are engaged in a quest for credibility, we are better able to make sense both of their different interests and of the process by which one kind of credit is transformed into another.[1]

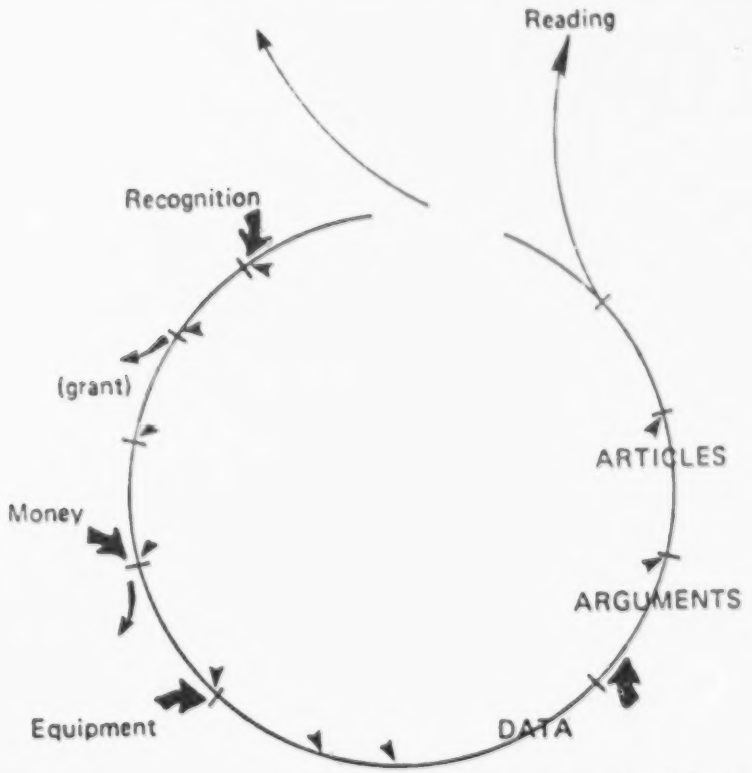
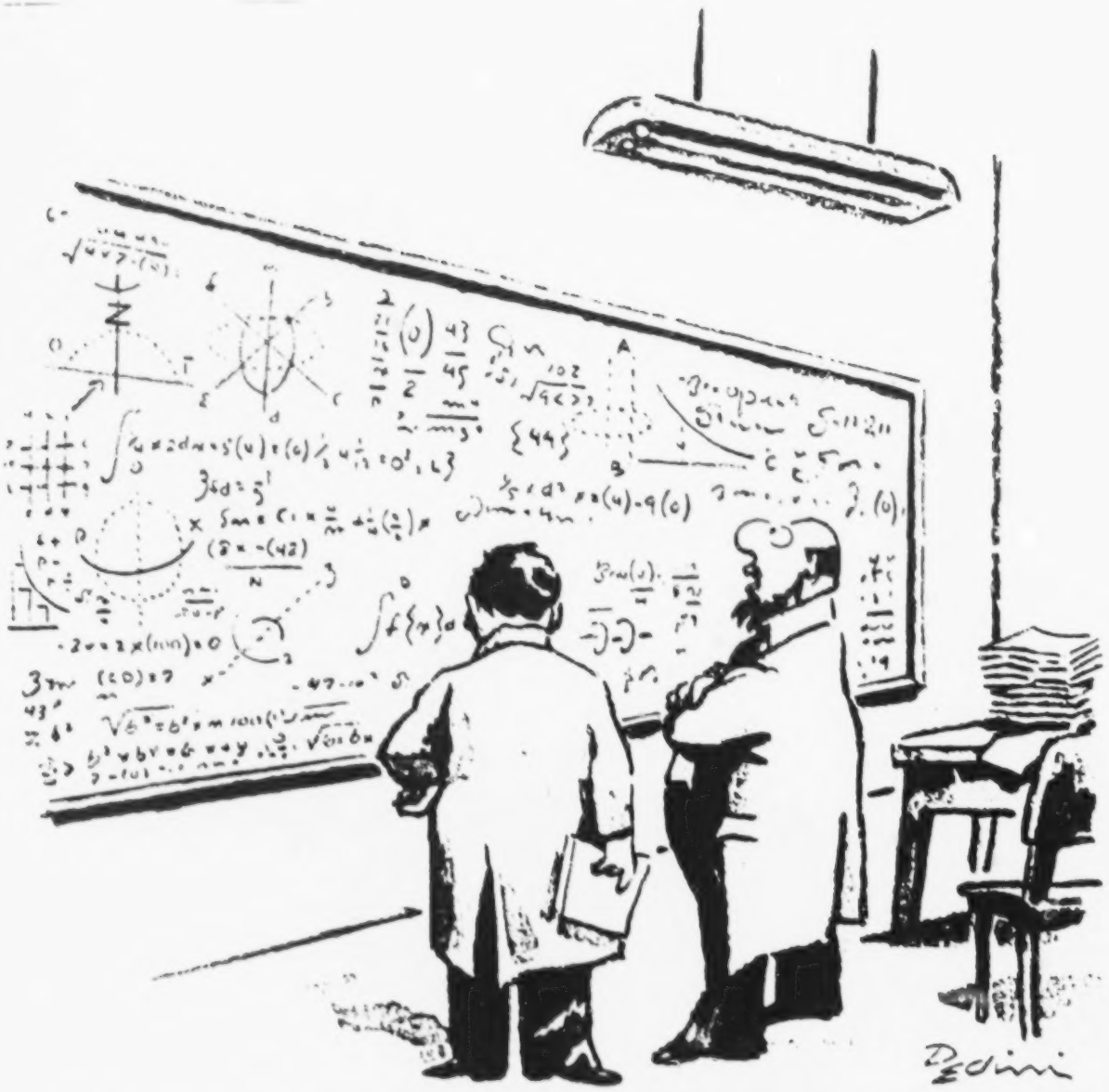


Figure 1 This figure represents the conversion between one type of capital and another which is necessary for a scientist to make a move in the scientific field. The diagram shows that the complete circle is the object of the present analysis, rather than any one particular section. As with monetary capital, the size and speed of conversion is the major criterion by which the efficiency of an operation is established. It should be noted that terms corresponding to different approaches (for example, economic and epistemological), are united in the phases of a single cycle.



"This is fine as far as it goes. From here on, it's who you know."